

Existence Predicate*

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Kant said that existence is not a predicate and Russell agreed, arguing that a sentence such as ‘The king of France exists’, which seems to attribute existence to the king of France, really has a logical form that is not reflected in the surface structure of the sentence at all. While the surface form of the sentence consists of a subject (the noun phrase ‘the king of France’) and a predicate (the verb phrase ‘exists’), the underlying logical form, according to Russell, is the formula given in (1). This formula obviously has no subject-predicate form and in fact has no single constituent that corresponds to the verb phrase ‘exists’ in the surface sentence.

$$(1) \exists x \forall y (Ky \leftrightarrow x = y)$$

The importance of Russell’s analysis becomes clear when we consider ‘The king of France does not exist’. If this sentence would attribute non-existence to the king it should entail that there is someone who does not exist, just as ‘Mary doesn’t like bananas’ entails that there is someone who doesn’t like bananas. Thus the idea that all sentences have subject-predicate form has led some philosophers (e.g. Meinong) to the view that there are objects that lack existence. This embarrassing position can be avoided once Russell’s analysis is accepted: if ‘The king of France does not exist’ is formalised as the negation of formula (1), no unwanted consequences follow.

There is, however, another side to the coin, for as Richard Montague has shown, we can have Russell’s analysis of definite descriptions and have subject-predicate form too, provided that we are prepared to allow lambdas and application into our logical language. With these means we can formalise ‘the king of France’ as $\lambda P \exists x (\forall y (Ky \leftrightarrow x = y) \wedge Px)$, a term that denotes the empty set if there is no unique king of France and the set of all sets that have

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the king as an element otherwise. If we apply this predicate of predicates to the existence predicate $\lambda z(z = z)$ we obtain formula (2).

$$(2) \lambda P \exists x (\forall y (Ky \leftrightarrow x = y) \wedge Px) \lambda z (z = z)$$

Note that the two main constituents of (2) correspond to the two main constituents of ‘The king of France exists’, the sentence it formalises. The logical translation of the noun phrase is now predicated of the translation of the verb phrase, the existence predicate $\lambda z(z = z)$. However, some manipulation shows that (2) is equivalent with (1), Russell’s formalisation of the sentence. Thus we can have our cake and eat it: have an existence predicate but avoid the position that there are things which are not.

Things become slightly more complicated when modal contexts are taken into account, for although there are no things that do not exist, objects that do in fact exist might not have existed, and things that do not in fact exist might have existed if the situation were different. Thus each possible situation comes with a domain of individuals that exist in that situation. In logics that allow quantification over possible situations as well as quantification over individuals it is therefore expedient to introduce an existence predicate E whose extension is always the set of things that exist in the given situation.